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Chaotic Behaviour of the Solutions of the Moore-Gibson-Thompson Equation

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Abstract: We study a third-order partial differential equation in the form

$$\tau u_{ttt} + \alpha u_{tt} - c^2 u_{xx} - b u_{xxt} = 0, \tag{1}$$

that corresponds to the one-dimensional version of the Moore-Gibson-Thompson equation arising in high-intensity ultrasound and linear vibrations of elastic structures. In contrast with the current literature on the subject, we show that when the critical parameter $\gamma := \alpha - \frac{\tau c^2}{b}$ is negative, the equation (1) admits an uniformly continuous, chaotic and topologically mixing semigroup on Banach spaces of Herzog's type.

Keywords: Acoustics, C_0-semigroups, Devaney chaos, hypercyclicity, Moore-Gibson-Thompson equation, sound propagation.
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1 Introduction

Basic problems in acoustics for the sound propagation are described in terms of the linear wave equation. For high wave amplitudes and intensities, new phenomena such as wave distortion and formation of shocks appear and the wave equation must be substituted by a nonlinear partial differential equation. The wide range of applications in bioengineering and industry of high intensity sound waves have encouraged investigations to go more deeply into this field of research [6,10,26,27,31,32].

The classical models of nonlinear acoustics are Kuznetsov's equation, the Westervelt's equation, and the Kokhlov-Zabolotskaya-Kuznetsov equation. Several initial boundary problems for these nonlinear second order in time partial differential equations have been considered very recently by Kaltenbacher and Lasiecka in collaboration with other authors, see for instance [22,23,24,25], and by Rozanova-Pierrat [28,29].

These models are of second order in time and characterized by the presence of a viscoelastic damping. The Kuznetsov's equation had been considered by many authors as the ”classical” acoustics equation. This equation for the velocity potential ψ is:

$$\psi_{tt} - c^2 \Delta \psi - \delta \Delta \psi_t = \left(\frac{1}{c^2} \frac{B}{2A} (\psi_t)^2 + |\nabla \psi|^2 \right)_t, \tag{2}$$

where c is the sound speed, δ is the diffusivity of the sound and B/A is the parameter of nonlinearity.

A complete model for a thermo-viscous flow in compressible fluid relate several physical quantities, such as the scalar sound velocity potential, the acoustic pressure, the mass density, the temperature, the heat flux and the entropy. If the heat flux is described by the classical Fourier transfer heat equation, the energy propagation has infinite speed. To avoid this paradox, other equations were considered to model the heat transfer in order to obtain a nonlinear acoustics wave equation. The Maxwell-Cattaneo equation combined with fluid physics equations leads to a third order in time partial differential equation model. This nonlinear equation is known as the

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Jordan-Moore-Gibson-Thompson equation:

$$\tau\psi_{ttt}+\psi_{tt}-c^2\Delta\psi-b\Delta\psi_t=\left(\frac{1}{c^2}\frac{B}{2A}(\psi_t)^2+|\nabla\psi|^2\right)_t, \quad (3)$$

where $b=\delta+\tau c^2$.

In this paper we consider the linearized version of this third order in time partial differential equation which is usually referred to as Moore-Gibson-Thompson equation:

$$\tau\psi_{ttt}+\psi_{tt}-c^2\Delta\psi-b\Delta\psi_t=0. \quad (4)$$

This equation displays a variety of dynamical behaviors for its solutions that heavily depends on the physical parameters in the equation.

Surprisingly, the linear equation (4) also arises in the study of the dynamics of linear vibrations of elastic structures. Bose and Gorain [5] proposed (4) as a model of vibrations of elastic structure in which the stress is not proportional to the strain.

We shall consider the one-dimensional version of equation (4)

$$\tau u_{ttt}+\alpha u_{tt}-c^2u_{xx}-bu_{xxt}=0, \quad (5)$$

with the initial conditions given by $u(0,x)=\varphi_1(x), u_t(0,x)=\varphi_2(x), u_{tt}(0,x)=\varphi_3(x)$, $x\in\mathbb{R}$ and where τ, α, c^2 and b are positive constants. Several stability and well-posedness properties of this third order equation, written even in a more general abstract way, have been studied in [11,15,16]. We point out that the third order in time model (5) exhibits very different qualitative behavior from the familiar second order complete equation $(\tau=0,\alpha>0)$. For third order in time equations, the critical parameter

$$\gamma\equiv\alpha-\frac{\tau c^2}{b},$$

plays a fundamental role in asymptotic behavior, energy estimates and regularity of solutions [27]. Indeed, all studies so far requires the positivity assumption $\gamma>0$. This is the common case considered in nonlinear acoustics, where $b\gamma$ is equal to the Lighthill's diffusivity of sound, which is always positive [20,21].

However, and excepting few results on the subject, the analysis of the behavior of (4) in case $\gamma\le 0$ remains largely open. Numerical calculations reveal that if the conditions $\gamma\ge 0, c>0$ do not hold and also for $\gamma>0, c=0$ the system (5) is unstable [24, Sec. 6]. In the same paper, it was also shown that equation (5) admits a strongly continuous group on Hilbert spaces, which is exponentially stable when $\gamma>0, c>0$ and not exponentially stable in the complementary region, see [24, Theorem 1.2].

Our main contribution in this paper gives new and interesting information about the behaviour of the equation (3) in the one-dimensional case and when the critical parameter γ is negative. Indeed, we prove the remarkable fact that for $\gamma<0$ the initial value problem (5) exhibits chaotic behaviour (Theorem 3.1). Our arguments are analitical rather than numercial and gives new insights about the dynamical behaviour in more general situations.

Previous effort on the understanding of dynamical behaviour of the solutions of linear partial differential equations like (5) can be found at the literature, see for instance [1,12,13]. For instance, the dynamical behaviour presented by the solutions of the heat equation was studied by Herzog [18] on certain spaces of analytic functions with certain growth control; on symmetric spaces of noncompact type in [19]; and on Damek-Ricci spaces[30].

A similar treatment to Herzog's approach was done in [8,17] for the hyperbolic heat transfer equation and the hyperbolic bioheat equation, a non-homogeneous version of the first one with internal heat sources [9]. The dynamical behaviour presented by the solutions of these equation becomes richer when the solutions are studied on certain spaces of analytic functions. On these spaces, phenomena such as chaos and topologically mixing are exhibited by the solutions of the hyperbolic heat and bioheat equations [8,17,9]. However, at the best of the knowledge of the authors, no study on dynamical behaviour -particularly chaos- has been done for the Moore-Gibson-Thomson equation (4). In this paper, we present first results in this direction for the one-dimensional setting, stimulating further analysis and work in the 2d and 3d situation, as well as in more general cases.

This paper is organized as follows: In the Section 2, we recall the definitions and tools needed for the statement of main result. In particular, we state a useful spectral criteria to determine Devaney Chaos for C_0-semigroups. Section 3 contains our main result (Theorem 3.1) which states that when the critical parameter γ is negative, the Moore-Gibson-Thomson equation admits chaos.

2 Preliminaries

We recall that a family $\{T_t\}_{t\ge 0}$ of linear and continuous operators on a Banach space X is said to be a *C_0-semigroup* if $T_0=Id$, $T_tT_s=T_{t+s}$ for all $t,s\ge 0$, and $\lim_{t\to s}T_tx=T_sx$ for all $x\in X$ and $s\ge 0$. Given a C_0-semigroup $\{T_t\}_{t\ge 0}$, it can be shown that an operator defined by $Ax:=\lim_{t\to 0}\frac{1}{t}(T_tx-x)$ exists on a dense subspace of X denoted by $D(A)$. Then A, or rather $(A,D(A))$, is called the *(infinitesimal) generator* of the semigroup. It can also be shown that the infinitesimal generator determines the semigroup uniquely. It is well-known that the generator A is bounded if and only if the semigroup is uniformly continuous, and in such case is expressed as $\{T_t\}_{t\ge 0}=\{e^{tA}\}_{t\ge 0}$, see for instance [14, Th. II.1.5].

The link between semigroups and differential equations is via the infinitesimal generator. The unique

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solution of the abstract Cauchy problem

$$\begin{cases} u_t = Au \\ u(0,x) = \varphi(x), \end{cases} \tag{6}$$

where A is the generator of a C_0-semigroup $\{T_t\}_{t\geq 0}$, is given by $u(t,x) = T_t\varphi(x)$ whenever $\varphi \in D(A)$. In that sense, $u(t,x)$ is called a *classical solution* of the abstract Cauchy problem (6).

In linear chaos, several notions can be considered when studying the linear dynamics of a C_0-semigroup $\{T_t\}_{t\geq 0}$, for further information regarding this topic see [17, Ch. 7].

We say that $\{T_t\}_{t\geq 0}$ is *hypercyclic* if there exists some $x \in X$ such that its orbit under the semigroup, $\{T_t x : t \geq 0\}$ is dense in X. A vector $x \in X$ is said to be a periodic point for $\{T_t\}_{t\geq 0}$ if there is some $t_0 \geq 0$ such that $T_{t_0}x = x$. A C_0-semigroup is said to be *Devaney chaotic* if it is hypercyclic and the set of periodic points is dense in X. A C_0-semigroup $\{T_t\}_{t\geq 0}$ is called topologically *mixing* if, for any pair U,V of nonempty open subsets of X, there exists some $t_0 > 0$ such that $T_t(U)\cap V \neq \emptyset$ for all $t \geq t_0$.

The following result is an useful consequence of the Hypercyclicity Criterion for C_0-semigroups [17]. Let X^* denote the dual space of X of linear and continuous functionals on X. We recall that by a weakly analytic function $f : U \to X$ on an open subset $U \subset \mathbb{C}$ we understand an X-valued function such that, for every $x^* \in X^*$, the complex valued function $z \mapsto \langle f(z), x^*\rangle$ is analytic on U. In the sequel, J is a nonempty index set.

Theorem 2.1. *([17, Theorem 7.30]) Let X be a complex separable Banach space and $\{T_t\}_{t\geq 0}$ a C_0-semigroup on X with generator $(A,D(A))$. Assume that there exists an open connected subset U and weakly analytic functions $f_j : U \to X,\ j \in J$, such that*

(i) $U \cap i\mathbb{R} \neq \emptyset$,
(ii) $f_j(\lambda) \in \ker(\lambda I - A)$ for every $\lambda \in U,\ j \in J$,
(iii) for any $x^* \in X^*$, if $\langle f_j(\lambda), x^*\rangle = 0$ for all $\lambda \in U$ and $j \in J$, then $x^* = 0$.

Then $\{T_t\}_{t\geq 0}$ is Devaney chaotic and topologically mixing.

This result can be compared with the Desch-Schappacher-Webb Criterion [12, Th 3.1], or any of its extensions [2,7]. Furthermore, either the Desch-Schappacher-Webb criterion or the Eigenvalue criterion for chaos imply distributional chaos, [3, Rem. 3.8], see also [4].

3 Devaney chaos for the Moore-Gibson-Thompson equation

We are going to consider the solution C_0-semigroup of the Moore-Gibson-Thompson equation on the following spaces:

$$H_\rho = \left\{ f : \mathbb{R} \to \mathbb{C}\,;\ f(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!}(\rho x)^n, (a_n)_{n\geq 0} \in c_0(\mathbb{N}_0) \right\} \tag{7}$$

with $\rho > 0$ and being $c_0(\mathbb{N}_0)$ the Banach space of all complex-valued sequences tending to 0. These are Banach spaces when endowed with the norm $||f|| = \sup_{n\geq 0}|a_n|$. In other words, the spaces H_ρ, $\rho > 0$, are Banach spaces of analytic functions with certain increasing control at infinity. The spaces H_ρ were introduced by Herzog [18] in connection with the study of dynamical behaviour of the heat equation. Observe that for any ρ fixed, the space H_ρ is naturally isomorphic to $c_0(\mathbb{N}_0)$. In particular, its dual H_ρ^* is isomorphic to the Banach space $l_1(\mathbb{N}_0)$ which consists of all complex-valued sequences (a_n) such that $\sum_{n=0}^{\infty}|a_n| < \infty$.

Example 1. Given $b \in \mathbb{C}$, the function $f(x) = \cosh(\sqrt{b}x)$ belongs to H_ρ if and only if $\rho^2 > |b|$. Analogously, the function $g(x) = e^{bx}$ belongs to H_ρ if and only if $\rho > |b|$.

Using the notation $u_1 = u$, $u_2 = \frac{\partial u}{\partial t}$, and $u_3 = \frac{\partial^2 u}{\partial t^2}$, the third order in time Cauchy problem in (5) can be rewritten as a first-order differential equation.

$$\frac{\partial}{\partial t}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & I & 0 \\ 0 & 0 & I \\ \frac{c^2}{\tau}\frac{\partial^2}{\partial x^2} & \frac{b}{\tau}\frac{\partial^2}{\partial x^2} & -\frac{\alpha I}{\tau} \end{pmatrix}}_{A}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \tag{8}$$

with the initial conditions given by $u_1(0,x) = \varphi_1(x), u_2(0,x) = \varphi_2(x), u_3(0,x) = \varphi_3(x), x \in \mathbb{R}$. Since for each $\rho > 0$ the operator $D : H_\rho \to H_\rho$ defined by $Df(x) = \frac{\partial^2}{\partial x^2}f(x)$ is clearly bounded, it follows that the operator-valued matrix A in (8) is a bounded linear operator on any space $X_\rho := H_\rho \oplus H_\rho \oplus H_\rho$, $\rho > 0$. Therefore, $\{e^{tA}\}_{t\geq 0}$ is an uniformly continuous semigroup on these spaces. Note that it contrast with the results in [24] where it was proved that A generates a strongly continuous group or semigroup in several phase spaces. Using the representation in (8) of the initial value problem, we can obtain the Devaney chaos of its solution semigroup under certain hypothesis on the parameters α, b, c, and τ. The proof follows ideas given in [8,17], and its an application of Theorem 2.1.

Theorem 3.1. *Let $\tau, b > 0$ and $\alpha \geq 0$ be given. Assume $\gamma := \alpha - \frac{\tau c^2}{b} < 0$. Then A generates a uniformly continuous, Devaney chaotic and topologically mixing semigroup on X_ρ for each $\rho^2 > \frac{2c^4\tau\alpha}{b^2(\tau c^2 - b\alpha)}$.*

Proof. Let U be the open disk of radius $r_0 = \frac{\rho^2 c^2 b^2}{2\tau c^4 + \rho^2 b^3}$ centered at zero. Then, condition 2.1.(i) holds directly. We define analytic functions of the form

$$\begin{aligned}\varphi_{\lambda,z_0,z_1}(x) &:= z_0\sum_{n=0}^{\infty}\frac{R_\lambda^n x^{2n}}{(2n)!} + z_1\sqrt{R_\lambda}\sum_{n=0}^{\infty}\frac{R_\lambda^n x^{2n+1}}{(2n+1)!} \\ &= z_0\cosh\left(\sqrt{R_\lambda}x\right) + z_1\sinh\left(\sqrt{R_\lambda}x\right), \ x \in \mathbb{R},\end{aligned} \tag{9}$$

where $\lambda \in \mathbb{C}, z_0, z_1 \in \mathbb{R}$, and $R_\lambda := \frac{\alpha\lambda^2+\tau\lambda^3}{c^2+b\lambda}$. So that, for any $\rho > 0$, the functions $f_{\lambda,z_0,z_1} : U \to X_\rho$, $z_0, z_1 \in \mathbb{R}$, defined by

$$f_{\lambda,z_0,z_1} := \begin{pmatrix} \varphi_{\lambda,z_0,z_1} \\ \lambda\varphi_{\lambda,z_0,z_1} \\ \lambda^2\varphi_{\lambda,z_0,z_1} \end{pmatrix} \tag{10}$$

satisfy that $Af_{\lambda,z_0,z_1} = \lambda f_{\lambda,z_0,z_1}$ for every $\lambda \in \mathbb{C}, z_0, z_1 \in \mathbb{R}$. Clearly, the functions f_{λ,z_0,z_1} are weakly analytic on U. Let us prove that f_{λ,z_0,z_1} are well defined on X_ρ for all $\rho > 0$ and $\lambda \in U$ and then condition of Theorem 2.1(ii) will hold. In order to do that, it is enough to prove that $\varphi_{\lambda,z_0,z_1} \in H_\rho$. Indeed, fix an arbitrary $\rho > 0$ and $\lambda \in U$. Since r_0 also fulfills the condition $r_0 < \frac{c^2}{b}$, then we have

$$\frac{|R_\lambda|}{\rho^2} < r_0^2\left(\frac{\alpha}{\rho^2(c^2-br_0)} + \frac{\tau r_0}{\rho^2(c^2-br_0)}\right). \tag{11}$$

On the one hand, due to the choice of r_0, we have

$$\frac{\tau r_0}{\rho^2(c^2-br_0)} = \frac{b^2}{2c^4}. \tag{12}$$

On the other hand, using (12) we get

$$\frac{\alpha}{\rho^2(c^2-br_0)} = \frac{\alpha\tau r_0}{\rho^2(c^2-br_0)}\cdot\frac{1}{\tau r_0} = \frac{\alpha b^2}{2c^4}\cdot\frac{1}{\tau r_0} \tag{13}$$

Combining equations (11),(12), and (13) with the condition $r_0 < \frac{c^2}{b}$ and $\gamma < 0$ we have

$$\frac{|R_\lambda|}{\rho^2} < r_0\frac{\alpha b^2}{2c^4\tau} + \frac{c^4}{b^2}\cdot\frac{b^2}{2c^4} + < \frac{\alpha b}{2c^2\tau} + \frac{1}{2} < \frac{1}{2} + \frac{1}{2} = 1. \tag{14}$$

Therefore, we can rewrite

$$\begin{aligned}\varphi_{\lambda,z_0,z_1}(x) &= z_0\cosh\left(\rho x\sqrt{\frac{R_\lambda}{\rho^2}}\right) + z_1\sinh\left(\rho x\sqrt{\frac{R_\lambda}{\rho^2}}\right) \\ &= \textstyle\sum_{n=0}^\infty a_n(\lambda)\frac{(\rho x)^n}{n!}, \quad x \in \mathbb{R},\end{aligned} \tag{15}$$

where $a_n(\lambda) := z_0\frac{R_\lambda^{n/2}}{\rho^n}$ if $n = 0, 2, 4, \dots$ and $a_n(\lambda) := z_1\sqrt{R_\lambda}\frac{R_\lambda^{(n-1)/2}}{\rho^n}$ if $n = 1, 3, 5, \dots$ Observe that by (14) the sequence $(a_n(\lambda))_{n\ge 0}$ belongs to $c_0(\mathbb{N}_0)$ for each $\lambda \in U$ fixed. This yields $\varphi_{\lambda,z_0,z_1} \in H_\rho$ for all $\rho > 0$.

Now, it only remains to see that condition of Theorem 2.1(iii) holds. Let $x^* = (x_1^*, x_2^*, x_3^*) \in X_\rho^*$ be fixed and denote $x_i^* = (x_{i,n}^*)_{n\ge 0}$ for $i = 1, 2, 3$. Since the space H_ρ is isomorphic to $c_0(\mathbb{N}_0)$, then X_ρ^* is isomorphic to $\ell_1(\mathbb{N}_0) \oplus \ell_1(\mathbb{N}_0) \oplus \ell_1(\mathbb{N}_0)$. Suppose that

$$\begin{aligned}0 &= \langle f_{\lambda,z_0,z_1}, x^*\rangle \\ &= \langle \varphi_{\lambda,z_0,z_1}, x_1^*\rangle + \langle \lambda\varphi_{\lambda,z_0,z_1}, x_2^*\rangle + \langle \lambda^2\varphi_{\lambda,z_0,z_1}, x_3^*\rangle\end{aligned} \tag{16}$$

for all $\lambda \in U$ and $z_0, z_1 \in \mathbb{R}$.

This last equation can be rewritten in terms of the isomorphic spaces $c_0(\mathbb{N}_0)$ and $\ell_1(\mathbb{N}_0)$ instead of H_ρ and H_ρ^*. So that, we have

$$0 = \sum_{n=0}^\infty a_n(\lambda)x_{1,n}^* + \lambda\sum_{n=0}^\infty a_n(\lambda)x_{2,n}^* + \lambda^2\sum_{n=0}^\infty a_n(\lambda)x_{3,n}^* \tag{17}$$

for all $\lambda \in U$ and $z_0, z_1 \in \mathbb{R}$.

It is clear that $0 \in U$ and $R_\lambda = 0$ when $\lambda = 0$. Observe that $\lambda_0 := -\frac{\alpha}{\tau} \in U$ because $\rho^2 > \frac{2c^4\tau\alpha}{b^2(\tau c^2-b\alpha)}$ by hypothesis. It can be seen that $R_{\lambda_0} = 0$. We also note that $\frac{\lambda^2}{R_\lambda}$ equals to $\frac{c^2}{\alpha}$ at $\lambda = 0$ and $\frac{\lambda^2}{\sqrt{R_\lambda}}$ equals to 0 at $\lambda = 0$.

Step 0: We evaluate in (17) at $\lambda = 0$ and we obtain $\langle f_{0,z_0,z_1}, x^*\rangle = a_0(0)x_{1,0}^* = z_0x_{1,0}^* = 0$ for all $z_0 \in \mathbb{R}$. Therefore $x_{1,0}^* = 0$.

Step 1: We evaluate in (17) at $\lambda = \lambda_0$ and we obtain

$$\lambda_0z_0x_{2,0}^* + \lambda_0^2z_0x_{3,0}^* = 0, \tag{18}$$

for all $z_0 \in \mathbb{R}$.

Step 2: We divide (17) by $\sqrt{R_\lambda}$ and we get

$$\begin{aligned}0 = &\tfrac{1}{\sqrt{R_\lambda}}\left(\textstyle\sum_{n=0}^\infty a_n(\lambda)x_{1,n}^* + \lambda\sum_{n=0}^\infty a_n(\lambda)x_{2,n}^*\right. \\ &\left. + \lambda^2\textstyle\sum_{n=0}^\infty a_n(\lambda)x_{3,n}^*\right).\end{aligned} \tag{19}$$

We evaluate in (19) at $\lambda = 0$. Then we obtain $\frac{1}{\rho}z_1x_{1,1}^* + \frac{c}{\sqrt{\alpha}}z_0x_{2,0}^* = 0$ for all $z_0, z_1 \in \mathbb{R}$. Therefore $x_{1,1}^* = x_{2,0}^* = 0$. In particular, we deduce from (18) that $x_{3,0}^* = 0$. So that, equation (19) can be reduced to

$$\begin{aligned}0 = &\tfrac{z_1}{\rho}\lambda x_{2,1}^* + \tfrac{z_1}{\rho}\lambda^2x_{3,1}^* + \tfrac{z_0}{\rho^2}\sqrt{R_\lambda}x_{1,2}^* \\ &+ \tfrac{z_0}{\rho^2}\sqrt{R_\lambda}\lambda x_{2,1}^* + \tfrac{z_0}{\rho^2}\sqrt{R_\lambda}\lambda x_{3,1}^* + \dots\end{aligned} \tag{20}$$

Step 3: We evaluate in (20) at $\lambda = \lambda_0$ and we obtain

$$\lambda_0z_1x_{2,1}^* + \lambda_0^2z_1x_{3,1}^* = 0, \tag{21}$$

for all $z_1 \in \mathbb{R}$.

Step 4: We divide (20) by $\sqrt{R_\lambda}$ and we get

$$\frac{z_1}{\rho}\frac{c}{\sqrt{\alpha}}x_{2,1}^* + \frac{z_0}{\rho^2}x_{1,2}^* = 0, \tag{22}$$

for all $z_0, z_1 \in \mathbb{R}$. Then $x_{1,2}^* = x_{2,1}^* = 0$, and coming back to (18) we get $x_{3,1}^* = 0$.

Proceeding inductively we can deduce that $x_{i,n}^* = 0$ for $i = 1, 2, 3$ and $n \in \mathbb{N}$, and then $x^* = 0$. By Theorem 2.1 the result follows.

*Remark.*The assumption $\gamma < 0$ in the statement of Theorem 3.1 directly yields $\frac{2c^4r\alpha}{b^2(rc^2-b\alpha)} > 0$

Finally, we point out that one can also study the dynamics of the solutions of the non-homogeneus Moore-Gibson-Thompson equation for certain internal sources appear, in the same way as this research is conducted in [9].

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